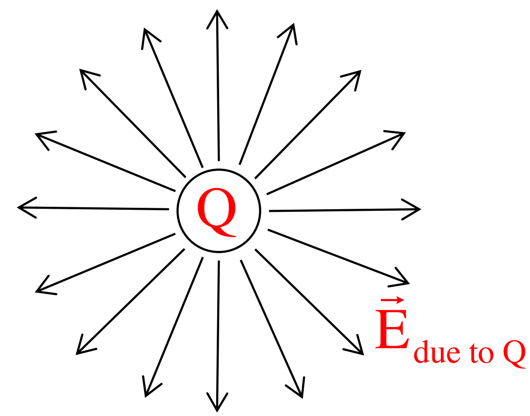
CHAPTER 25:
Electric Potentials and Energy Considerations

courtesy of
Mr. White

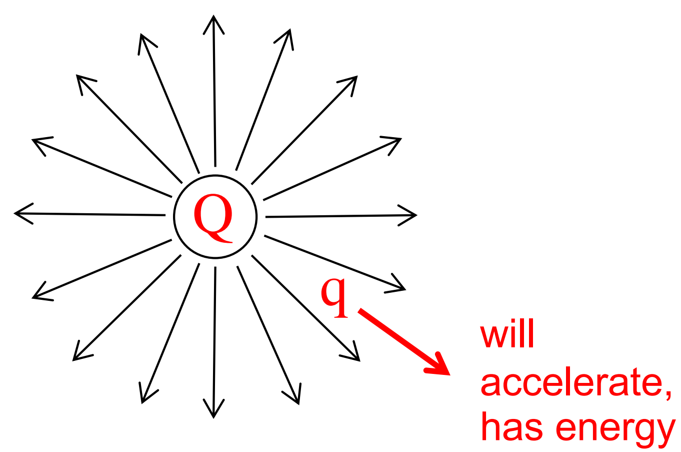
Electric Potential Fields



You should now be comfortable with the idea that a *charge configuration* will produce an electrical disturbance in its vicinity, and that knowing how much *force per unit charge* is provided to the region around the field-producing charge (whether there be a secondary charge is in the region experiencing the force or not) in the form of an *electric field* is a useful idea to entertain.

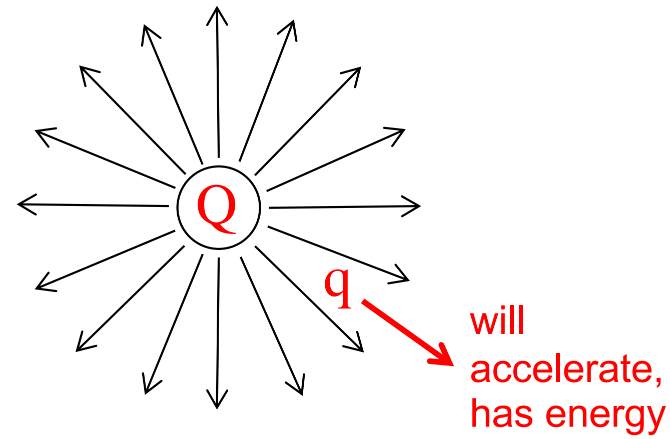
It's time to consider another related field, one associated with *energy*.

If we release a test charge *q* (or *any* charge, for that matter) in the electrical disturbance generated by our *field-producing charge*, the test charge WILL ACCELERATE.



Why will the test charge accelerate?

Because there is *POTENTIAL ENERGY* available to the test charge as it sits in the field.

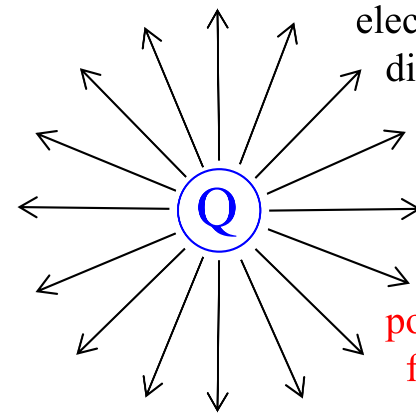


We could measure the amount of potential energy the test charge has, but that would be quite limiting (the information would be applicable only to that particular test charge).

The clever thing to do would be to mimic what we did with *electric fields*. We could measure the test charge's *potential energy* while in the field at a particular point, then divide by the size of the test charge to determine how much *POTENTIAL ENERGY PER UNIT CHARGE* is *AVAILABLE at the point* (whether the test charge is there to feel the effect or not).

This quantity, with units of *joules per coulomb* (or volts), is called the *ABSOLUTE ELECTRIC POTENTIAL* at the point of interest.

An ELECTRIC POTENTIAL FIELD, measuring the amount of *potential energy per unit charge AVAILABLE* at all points in the region of a *field-producing* charge, can be (and is) associated with *any* charge configuration.



An ELECTRIC POTENTIAL FIELD exists wherever there is charge (and, for that matter, wherever there is an *electric field*). For the potential fields to exist, there doesn't need to be present a *secondary charge* to feel the effect. And because *voltage-flds* tell us how much energy is available *PER UNIT CHARGE* at a point, the *electric potential field V* is defined as:

$$V = U/q$$

Important note: As an *absolute electric potential* is a function of the charge *q* that generates the field, a negative charge will produce a NEGATIVE *absolute electric potential* and a positive charge will produce a POSITIVE *absolute electric potential*!

Work and Electric-Potential (Voltage) Fields

Note: An *absolute electric potential field* is a modified *potential energy* field.

Everything you can do with *energy considerations*, you can do with *electric potential functions*:

Just as the work done on a body moving from one point to another in a *conservative force field* equals $W = -\Delta U$, we can use the definition of absolute electric potential to write:

$$W = -\Delta U = -q\Delta V$$

$$\Rightarrow \quad \frac{W}{q} = -\Delta V$$

and

Apparently, if you know the *voltage difference* between two points, you know how much *work per unit charge* AND *potential energy change per unit charge* the field has available between the two points.

$$W = -\Delta U = -q\Delta V$$

$$\Rightarrow \quad \frac{\Delta U}{q} = \Delta V$$

Example 5: How much work does a field do on a moving 2 C charge if the *potential difference* between its beginning and end points is 7 volts?

$$\frac{W}{q} = -\Delta V \quad \Rightarrow \quad W = -q\Delta V$$

$$= -(2\text{ C})(7\text{ J/C}) = -14\text{ J}$$

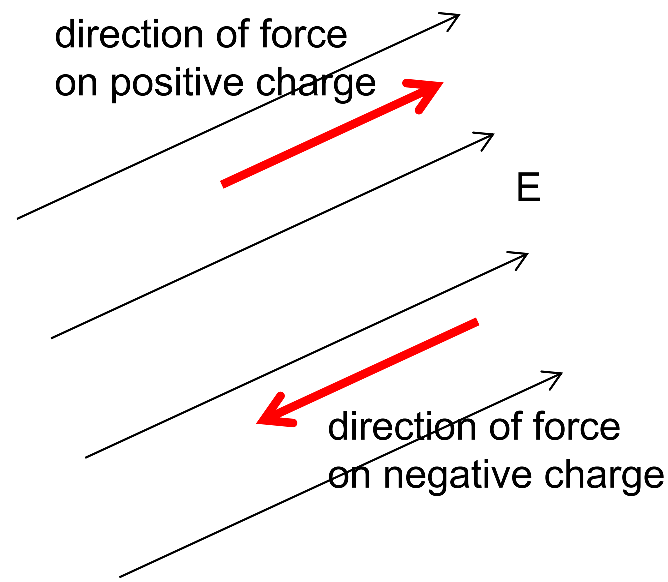
Subtlety About Conservation of Energy Equation

What electric field will be needed to stop an electron with kinetic energy K in distance d. In what direction should the the field be, opposite the direction of the electron's motion or with the electron's motion?

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$
$$K \quad + \quad 0 \quad + \vec{F}_e \bullet \vec{d} \quad = \quad 0 \quad + \quad 0$$
$$\Rightarrow \quad K + \left(q\vec{E}\right) \bullet \vec{d} = 0$$

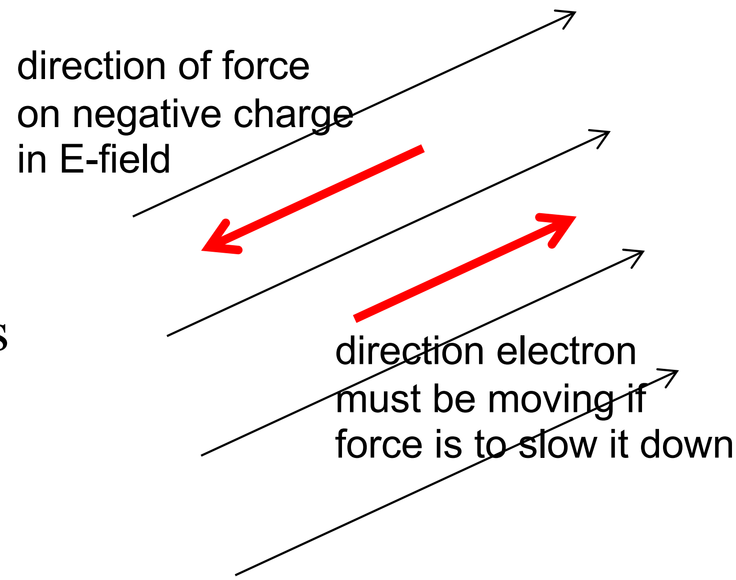
This dot product is a little tricky. To determine the angle between the electric field E and d, where d is in the direction of the velocity vector, we need to think a little bit about how an electron behaves in an electric field, and about what THIS electron is doing in this problem.

The electric force on an electron in an electric field will be exactly opposite that of the electric force on a positive charge. (Remember, the direction of an electric field is defined as the direction of force on a positive charge in the field, so the direction of force on an electron will be OPPOSITE the electric field direction.)



If the electron is moving OPPOSITE the direction of an electric field (i.e., in the direction of the force on it), it will speed up. If the electron is to slow down, it must be moving WITH the electric field.

That means the angle between E and d must be zero and the dot product will, due to its cosine factor, be positive.

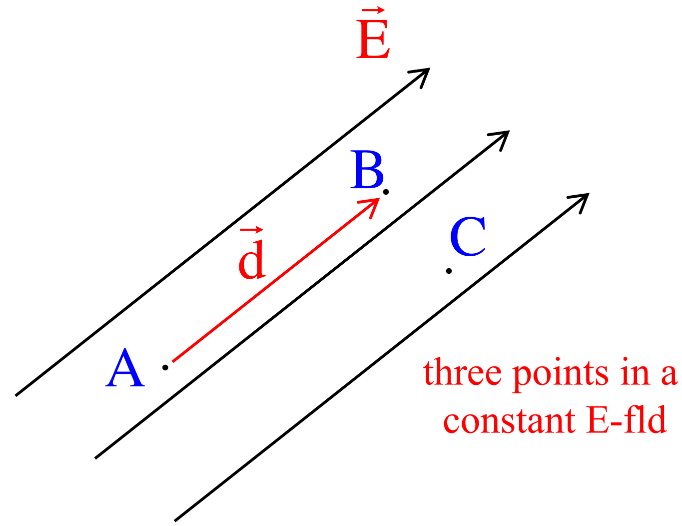


But we KNOW the work being done on the electron must be negative as it's slowing down, so where does the negative sign come from? It comes from the fact that the electron feeling the force is negative. That is, q = -e = -1.6x10^-19 for an electron. That is where the negative sign comes from.
Mathematically, this can be written as:

$$K + (-eE)d\cos(0^\circ) = 0$$

$$\Rightarrow \quad E = \frac{K}{ed}$$

Observations



Example 7: Points A, B and C are identified in a constant *electric field* as shown in the sketch.

a.) Which point has the GREATER *absolute electric potential*? (That is, do *electric fields* run from *higher voltage* to *lower*, or vice versa?)

Traversing from A to B, so $\vec{d}$ points along the line of $\vec{E}$, the *dot product* in our relationship falls out as:

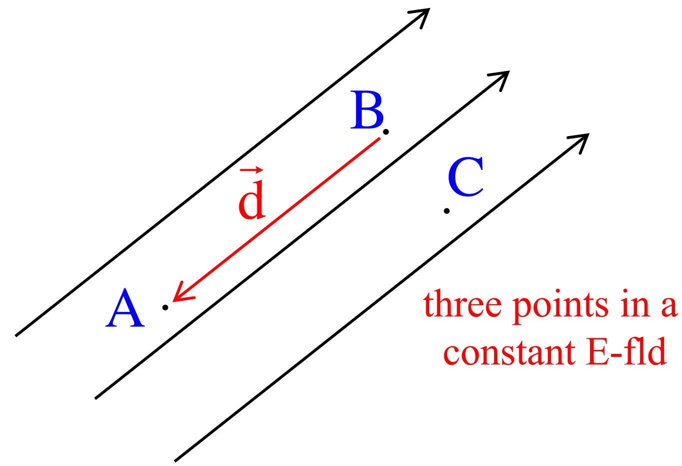
$$\vec{E} \bullet \vec{d} = -\Delta V$$

$$\Rightarrow \left|\vec{E}\right|\left|\vec{d}\right|\overset{1}{\cancel{\cos 0^\circ}} = -\left(V_B - V_A\right)$$

By observation, the left-side of the equation is positive (two magnitudes multiplied together), so the right-side must also be positive. For this to be true, V_A must be larger than V_B .

IMPORTANT OBSERVATION: This means that *ELECTRIC FIELDS* migrate from HIGHER *ELECTRIC POTENTIAL* to LOWER.

b.) Assume the *electric potential* at A is $V_A = 11$ volts and the *electric potential* at B is $V_B = 5$ volts. If the *distance* between the two points is 2 meters, derive an expression for the magnitude of the *electric field*.

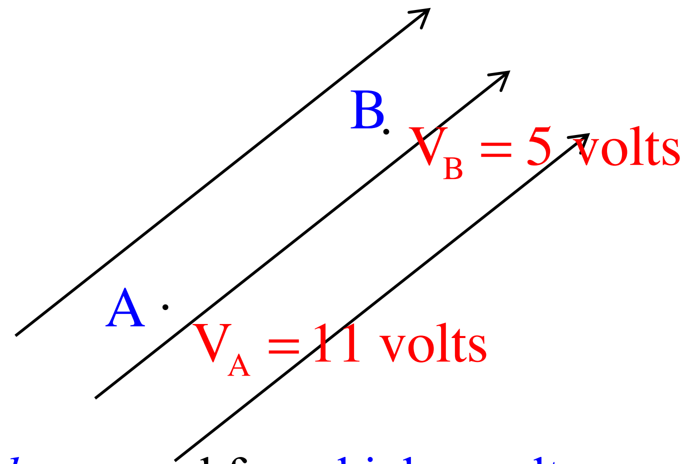


This time, to point out how the angle works, we will traverse from B to A. Noticing that now the angle between $\vec{d}$ and $\vec{E}$ is 180°, we can write:

$$
\begin{aligned}
&\vec{E} \bullet \vec{d} = -\Delta V \\
&\Rightarrow \quad |\vec{E}| \; |\vec{d}| \; \overset{-1}{\cancel{\cos 180^\circ}} = -(V_A - V_B) \\
&\quad -|\vec{E}|(3\text{m}) \qquad\qquad = -(11\text{v} - 5\text{v}) \\
&\qquad\qquad \Rightarrow \quad |\vec{E}| = 2 \text{ V/m}
\end{aligned}
$$

IMPORTANT SIDE POINT: The unit for *ELECTRIC FIELDS* is *newtons per coulomb*, but it is also, apparently, *volts per meter.*

c.) A positive charge Q=1C and mass *m=1 kg* moves naturally along the *E-fld* lines.



i.) Is the charge moving from *higher electrical potential* *to* *lower*, or *lower electric potential to higher*?

This has nothing to do with the charge. *Electric fields* proceed from higher voltage to lower, so it's doing the former.

ii.) Is the charge moving from higher *potential energy* to lower, or lower *potential energy* to higher?

This has EVERYTHING to do with the charge. POSITIVE CHARGES naturally move from *higher to lower voltage* along *E-fld* lines (being by definition the direction a positive charge would naturally accelerate), so it is moving from *higher* to lower potential energy.

iii.) If Q's initial velocity was 3 m/s at A, what is its velocity at B? (Note that the voltages have been put on the sketch.)

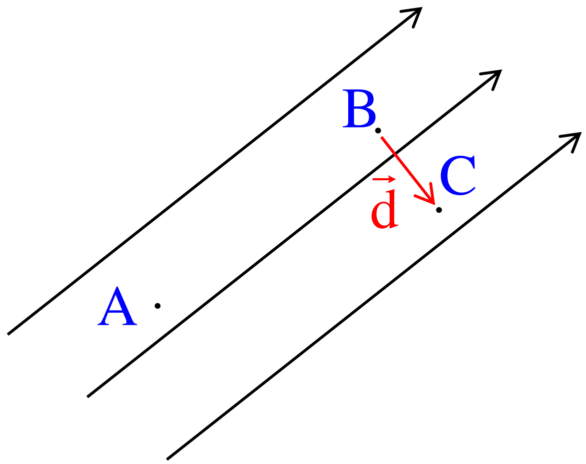
$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$\tfrac{1}{2} m v_A{}^2 + (qV_A) + 0 = \tfrac{1}{2} m v_B{}^2 + (qV_B)$$

$$\tfrac{1}{2}(1)(3)^2 + (1)(11) = \tfrac{1}{2}(1) v_B{}^2 + (1)(5)$$

$$\Rightarrow \quad v_B = 4.58 \text{ m/s}$$

d.) Given the *electric potential* at B is $V_B = 5 \text{ volts}$ and the *electric field,* as calculated in the previous part, is $|\vec{E}| = 2 \text{ V/m}$, what is the *voltage* (i.e., the *electric potential*) at C, assuming the distance between B and C is .5 meters?



This is slightly tricky. Define $\vec{d}$ on your sketch as shown. Notice that the angle between $\vec{d}$ and $\vec{E}$ is 90°. With that, we write:

$$\vec{E} \bullet \vec{d} = -\Delta V$$

$$\Rightarrow \quad |\vec{E}| \; |\vec{d}| \; \overbrace{\cancel{\cos 90^\circ}}^{0} = -(V_c - V_B)$$

$$\Rightarrow \quad 0 = -(V_C - V_B)$$

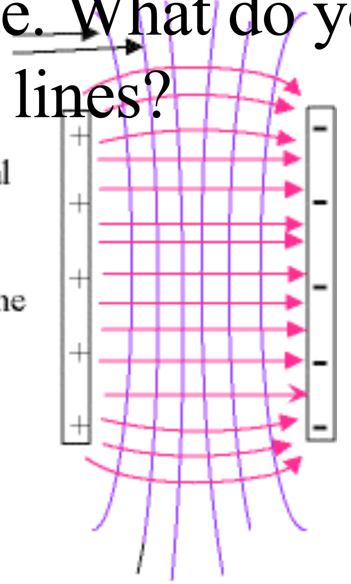
$$\Rightarrow \quad V_c = V_B = 5 \text{ V/m}$$

IMPORTANT POINT: An *EQUIPOTENTIAL LINE* is a line upon which every point has the same electrical potential. Points B and C are on the *5-volt equipotential line.*

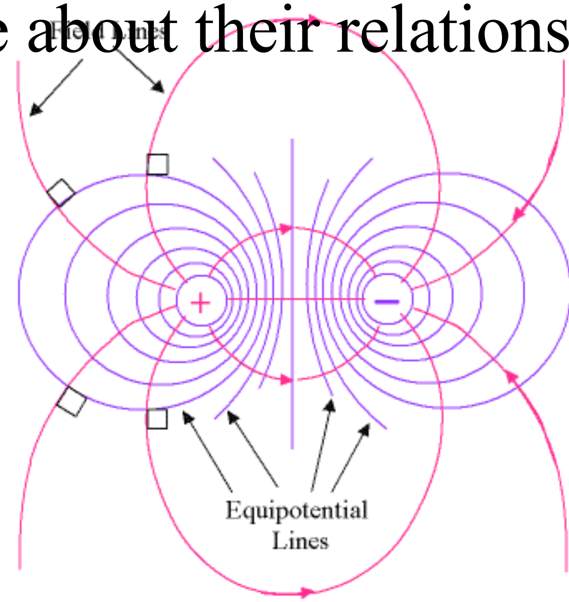
IMPORTANT POINT: *Equipotential lines* are ALWAYS perpendicular to *electric field lines*.

Equipotential Lines

- Equipotential lines are a way to represent lines of equal potential - that is, lines along which a charge would move with constant speed (no change in E).
- Two examples are shown here. The equipotential lines are in blue. What do you notice about their relationship to E field lines?



The horizontal and parallel lines are the field lines. The electric field is essentially uniform in between the plates.

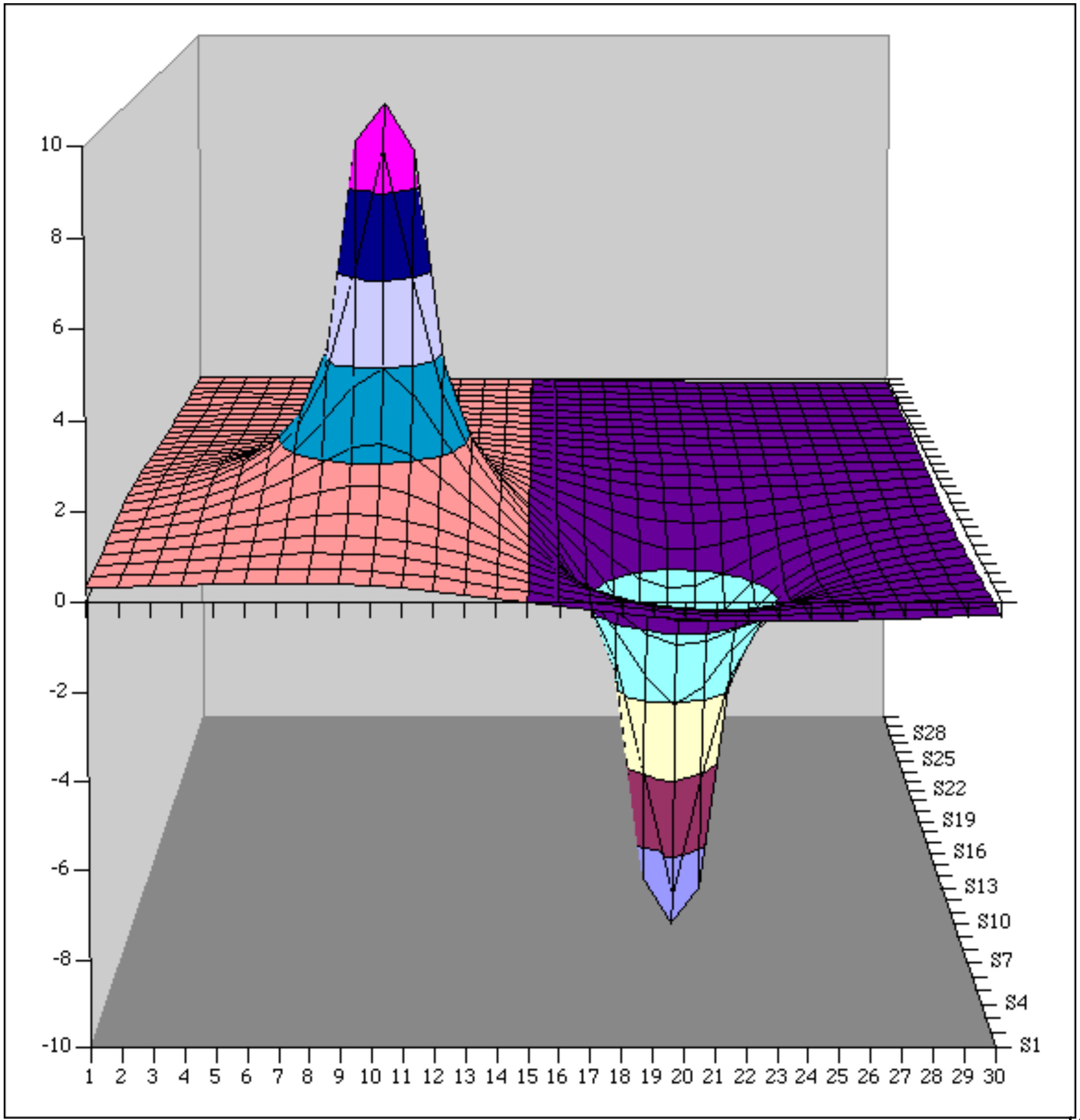


Important note: potential is represented by equipotential lines that <u>cross</u> E field lines. There is no such thing as an "electric potential field line" - it's not a field!

Electric Potential contours (courtesy of Mr. White)

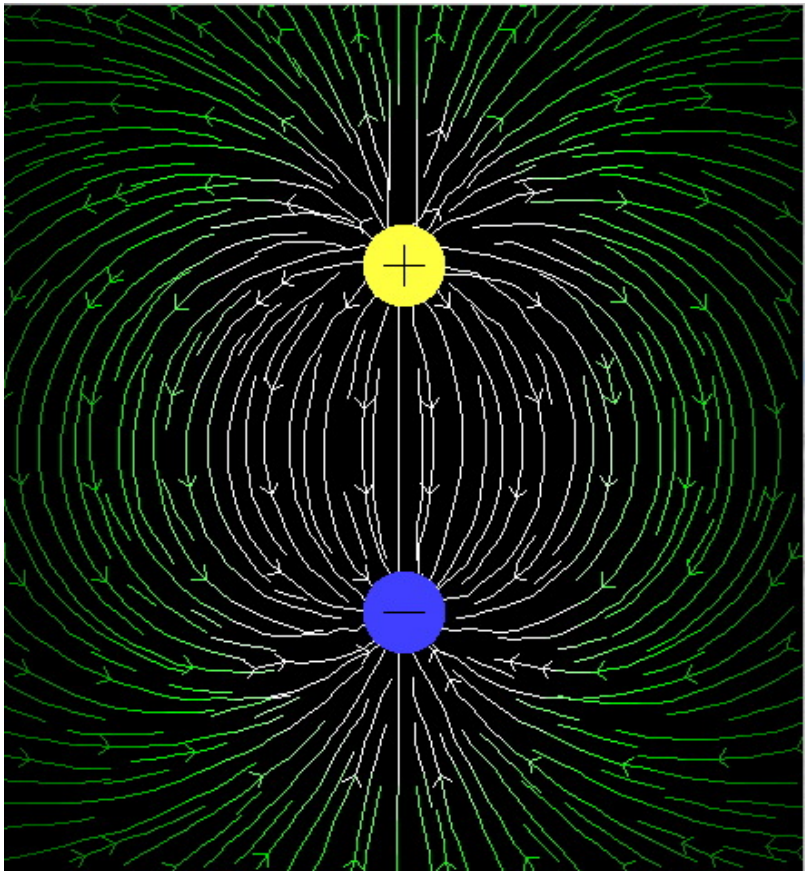
What are the potentials of each of the charges shown here? What do these potential values represent?

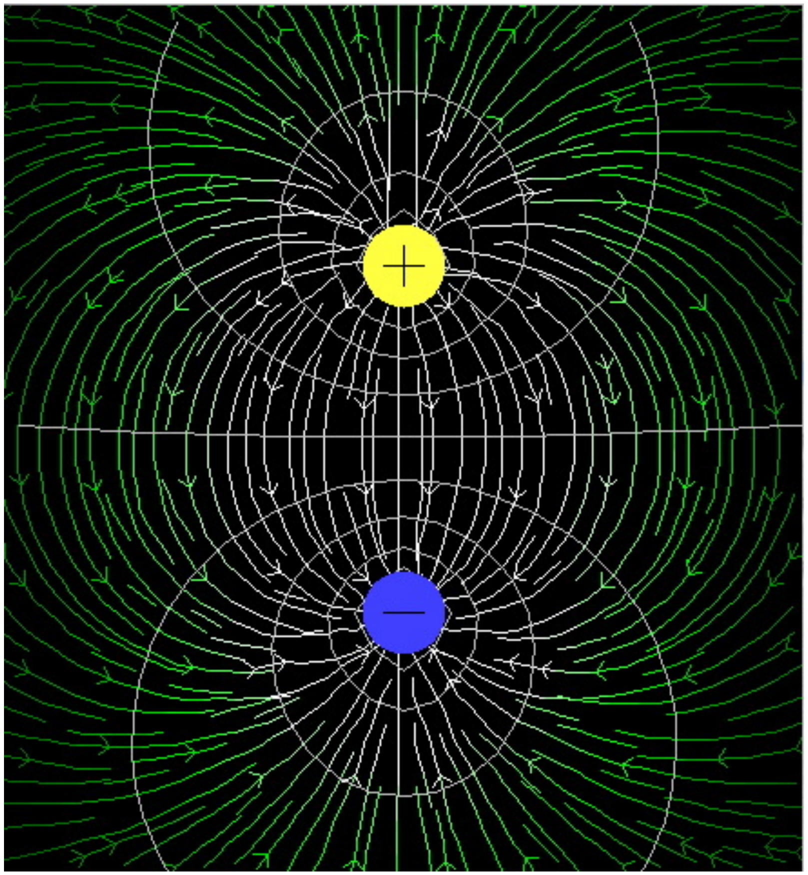
$$V_{net} = \sum_i k\frac{q_i}{r_i}$$



Equipotential Lines
(courtesy of Mr. White)

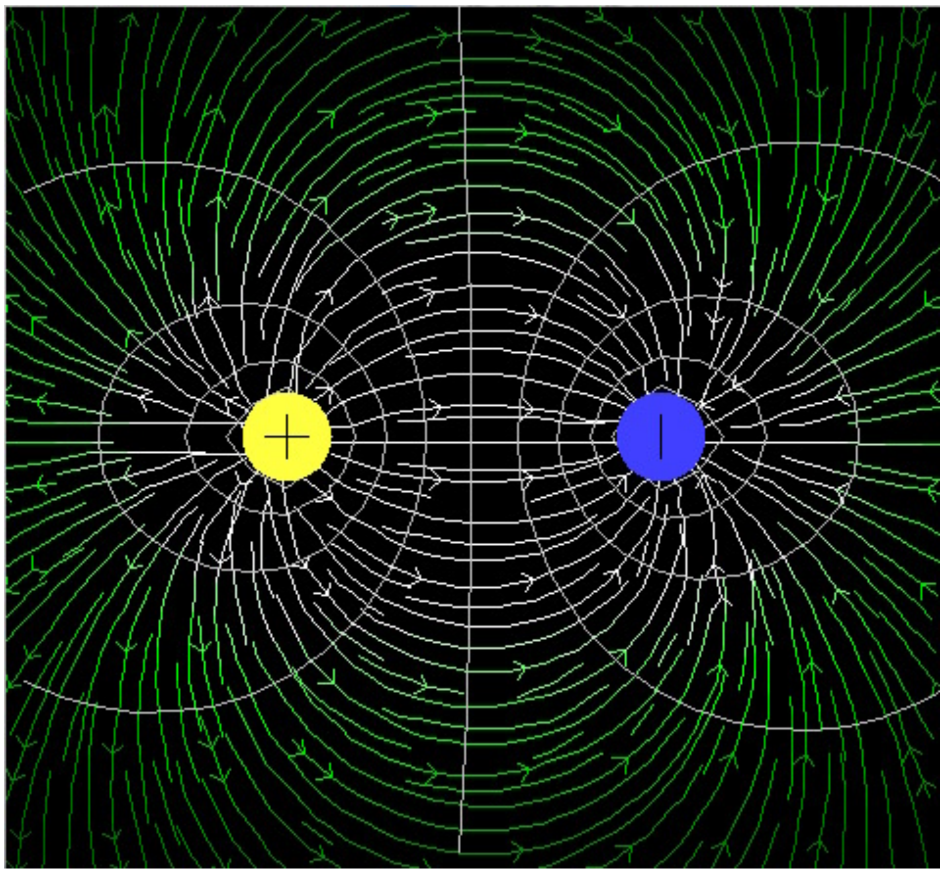
... are related to Electric Field lines. How?

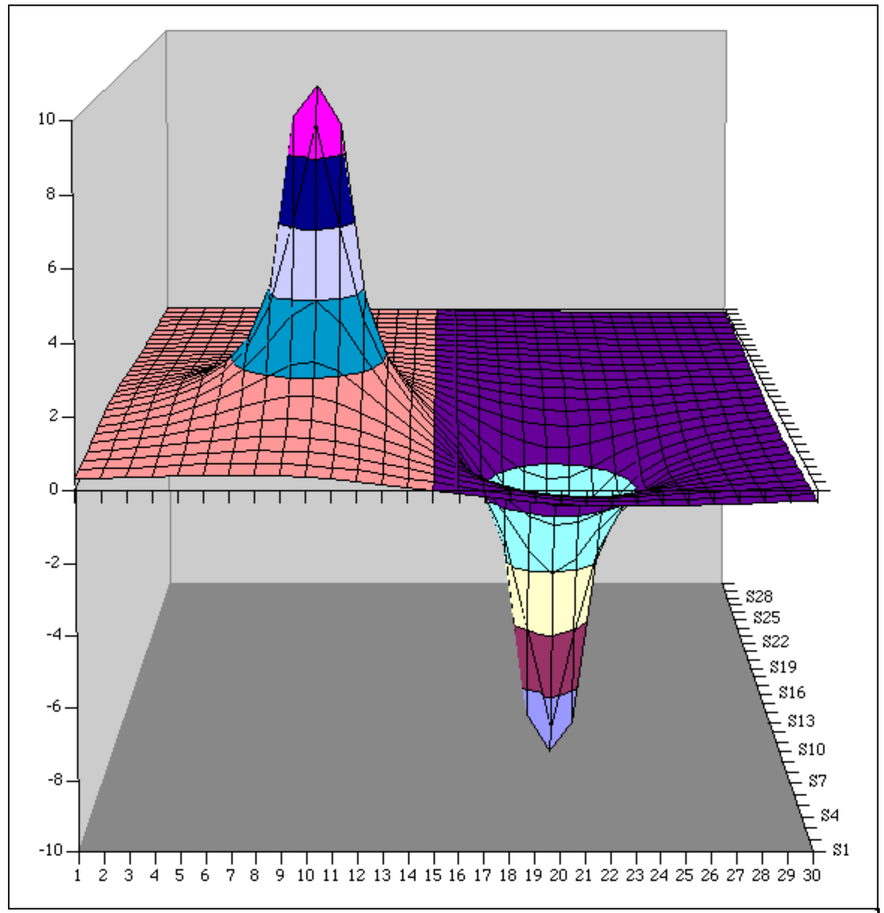




Equipotential Lines
(courtesy of Mr. White)

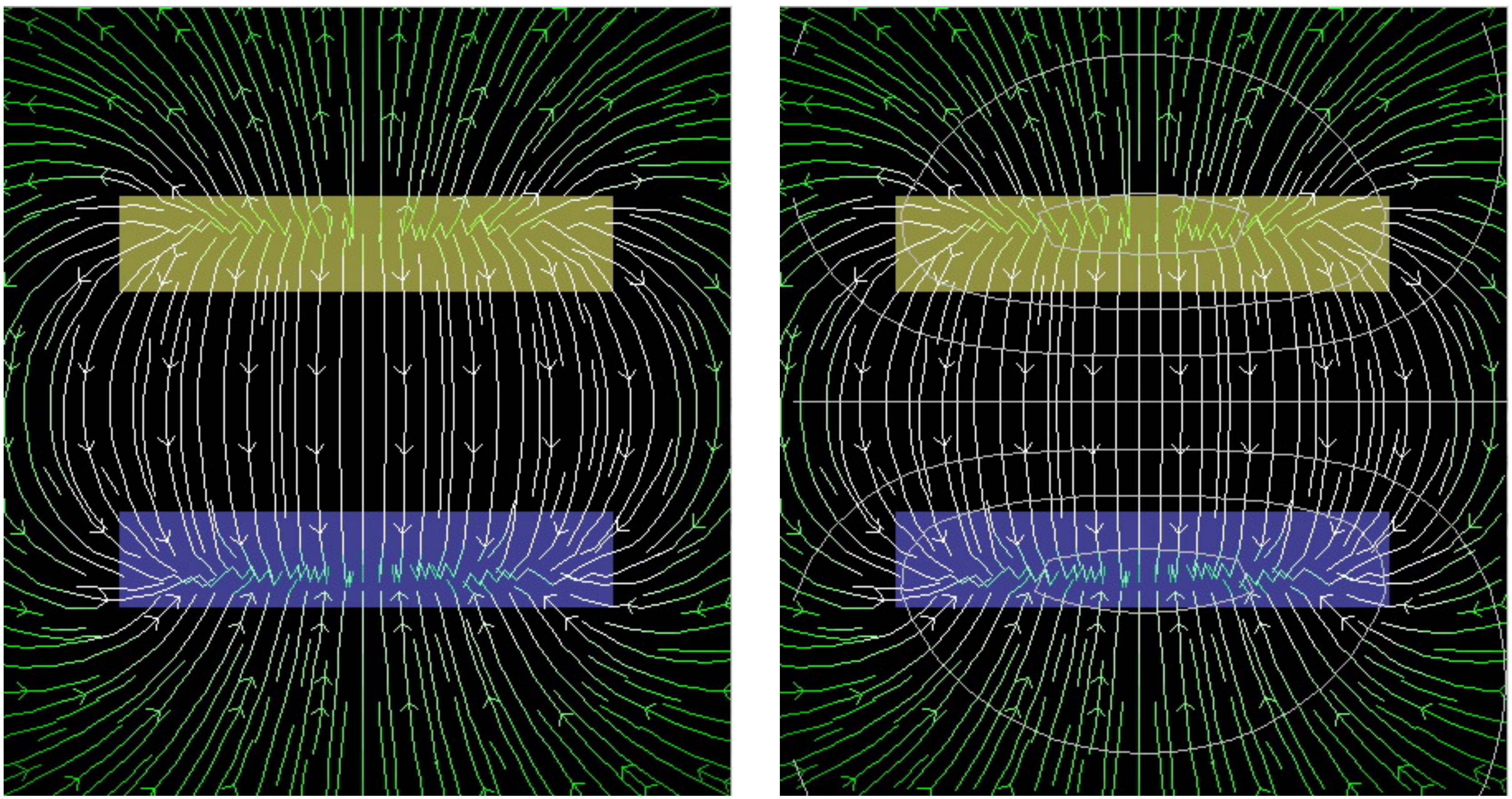
... reveal areas of higher vs. lower electric potential.





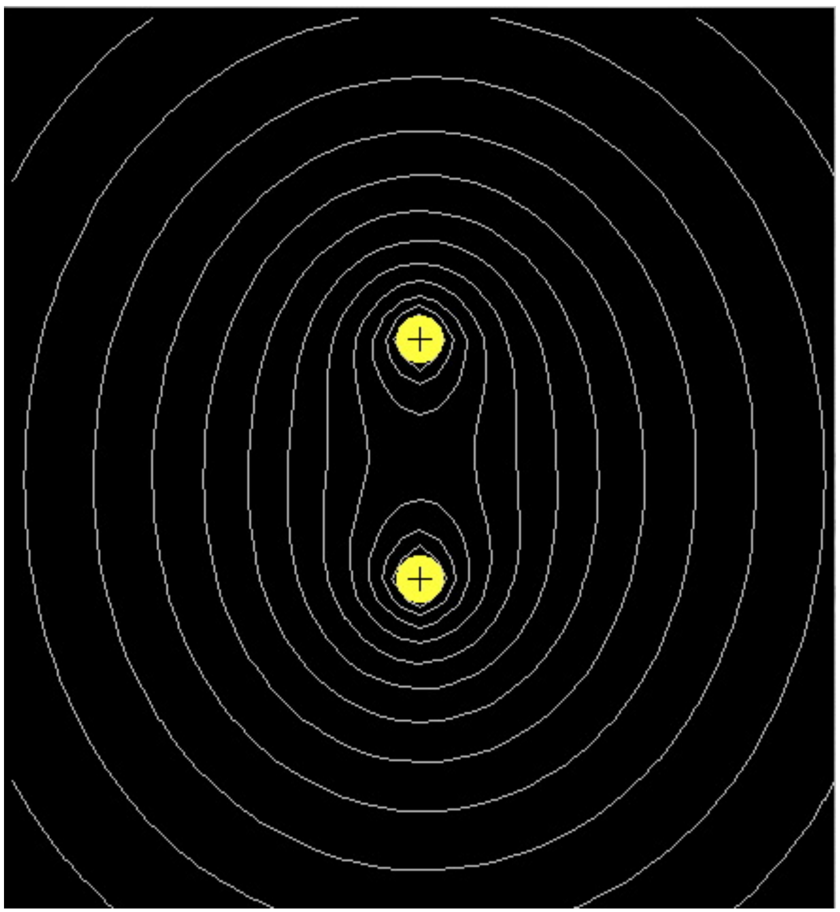
Example 8: (courtesy of Mr. White)

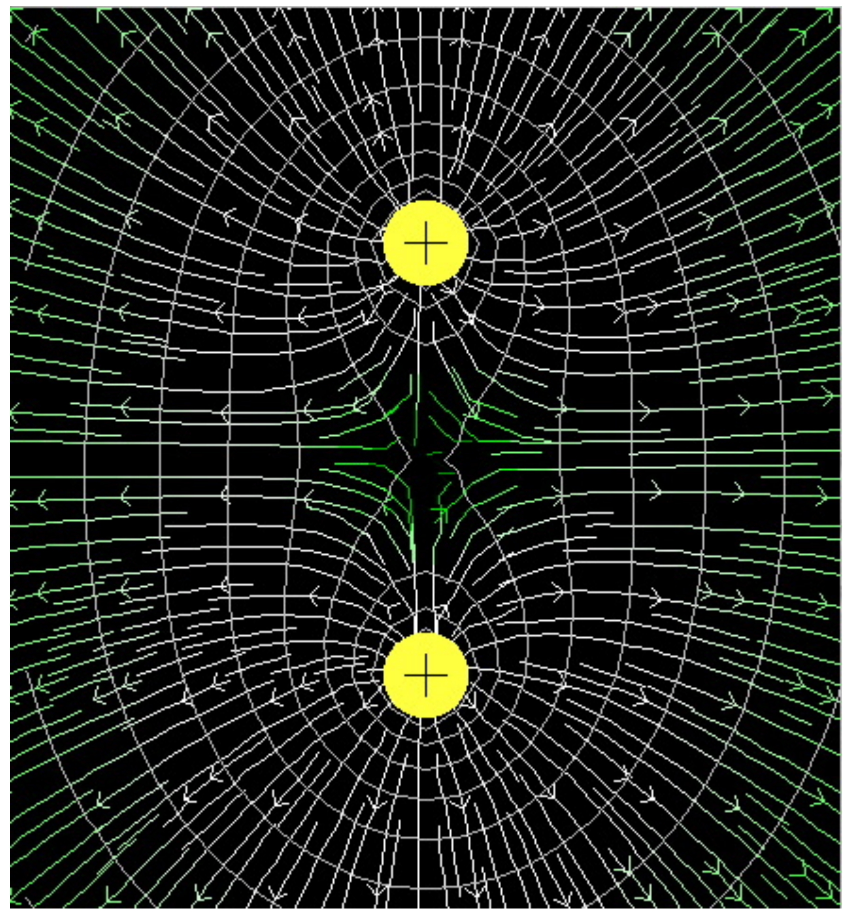
Draw appropriate equipotentials for this electric field.



Example 9: (courtesy of Mr. White)

Draw appropriate field lines for these equipotentials.





Things to Note About Electrical potential energy

Some things to note about this definition:

- *The equation* $\Delta U_e = -W_{ab} = -qE_x\Delta x$ is only valid for a <u>uniform electric field</u>, for a particle <u>displacing along a given axis</u>
- *This equation* is valid for both positive and negative charges!
  - *Moving in the direction* of the electric field yields a <u>drop</u> in potential energy for a positive charge, but a <u>gain</u> in potential energy for a negative charge.
- *In this equation*, **the sign of q and sign of E should be included in the calculation**
  - *For a negative charge*, you need a negative sign included in your calculation.
  - *E is usually positive*, but say you have a problem that asks what would happen if the field reversed direction - you would just put a negative sign in front of E as well.
- *Electrical potential energy* depends on both the charge and the field (like gravitational potential energy, mgh, this is now qEd)

Also remember the work-energy theorem: $W = \Delta KE$

- *If you know* the change in electrical potential energy, you can find its change in kinetic energy!

Quick check:

How much potential energy does a 2 C charge have at a point where the absolute electrical potential is 3 J/C?

$V = U/q$ so $U = qV = (2\text{ C})(3\text{ J/C}) = 6\text{ J}$

How much potential energy does a -2 C charge have at that same point?

$V = U/q$ so $U = qV = (-2\text{ C})(3\text{ J/C}) = -6\text{ J}$

How much potential energy does a -2 C charge have at a point where the absolute electrical potential is -3 V?

$V = U/q$ so $U = qV = (-2\text{ C})(-3\text{ J/C}) = 6\text{ J}$

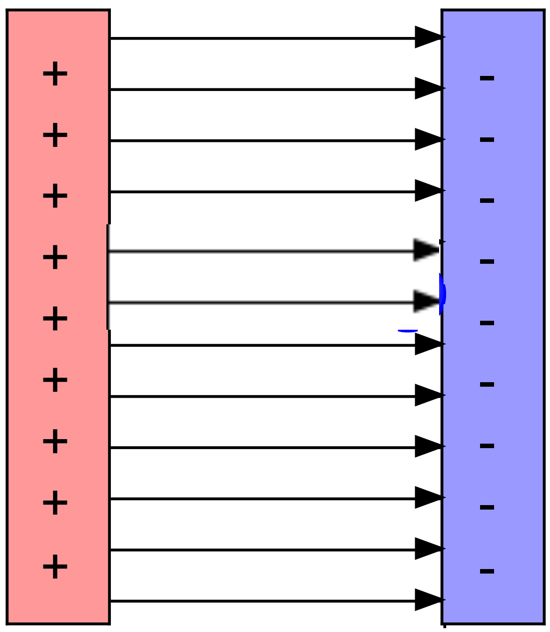
Potential vs potential energy

Electric field lines always point from high potential to low potential --> this is regardless of whether there is a positive, negative, or no charge present at that location at any time

Where high U_E is, however, depends on whether the charge feeling the effect is positive or negative charge!

- *The high U_E position* for a **positive** charge is the high V plate -- it wants to go towards the low voltage (negative) plate
- *The high U_E position* for a **negative** charge is the low V plate -- it wants to go towards the high V (positive) plate

High V
Low V

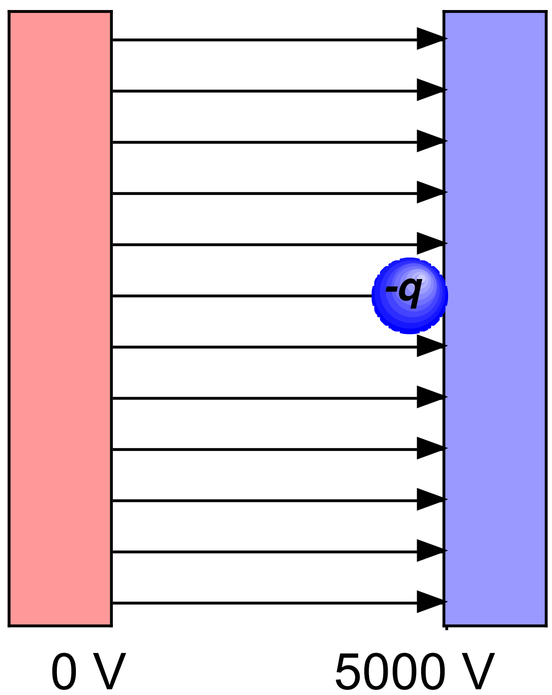


High U_E for a + charge

High U_E for a - charge

Another quick example

An electron (e^-) in a TV picture tube is accelerated from rest through a potential difference of 5000 V. The mass of an electron is 9.11 x 10^{-31} kg.



What is the change in U of the electron?

$$\Delta U = q\Delta V$$
$$\Delta U = (-1.602\text{x}10^{-19}\ \text{C})(5000\text{V} - 0\text{V})$$
$$\Delta U = -8\text{x}10^{16}\ \text{J} \quad (= 5000\text{eV})^*$$

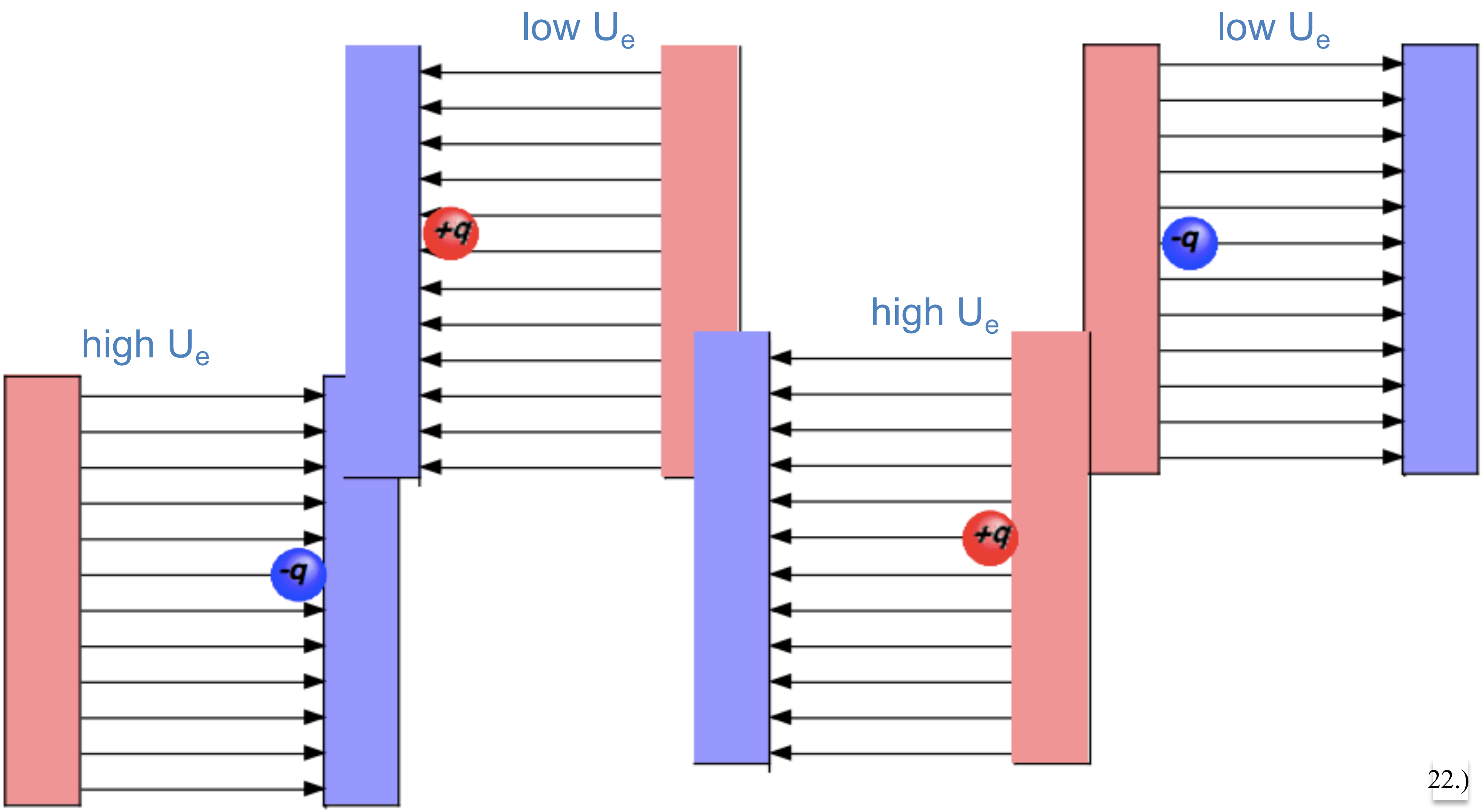
What is the final speed of the electron?

$$W = -\Delta U = -(-8\text{x}10^{16}\ \text{J}) = \Delta KE$$
$$½mv_f^2 - 0 = 8\text{x}10^{16}\ \text{J}$$
$$v_f = \sqrt{\frac{2(8\text{x}10^{16}\ \text{J})}{9.11\text{x}10^{-31}\text{kg}}} = 4.05\text{x}10^7\ \text{m/s}$$

**Fletch's note:* An electron-volt (eV) is defined as the amount of energy an electron picks up when accelerated through a *1 volt electrical potential difference*.

Visual practice *(thanks Mr. White!)*

Is the electrical charge in each of these situations in a position of *high* U_e or *low* U_e?



Quick practice

If an electron is released from rest in a uniform electric field, the potential energy of the charge-field system

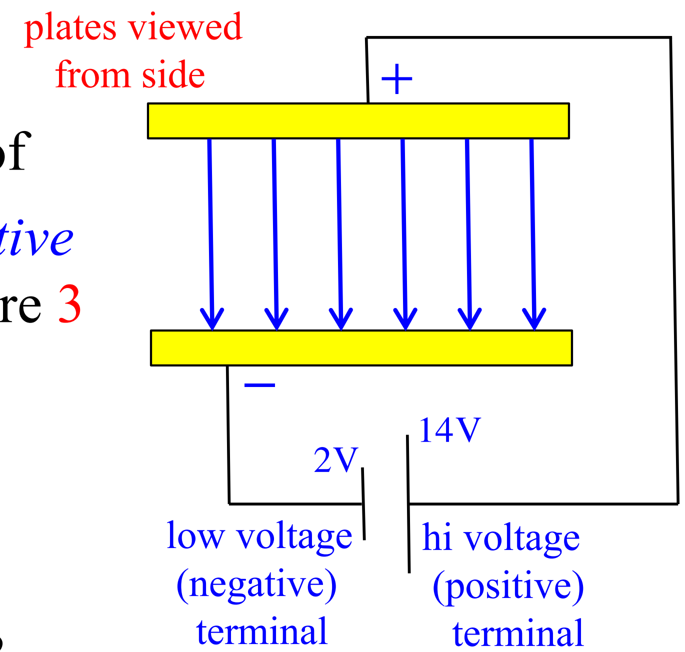
- (a) increases (b) decreases (c) stays the same Explain!

  When released from rest, the electron will accelerate in the direction opposite the electric field, due to the electric force on it by that field. This means electrical potential energy is converted to kinetic energy, and the potential energy will **decrease**.

True or false: If a proton and electron both move through the same displacement in an electric field, the change in PE for the proton must be equal in magnitude and opposite in sign to the change in PE for the electron.

  True: $\Delta U = -qEd$, and since both have the same magnitude charge, same displacement, and are in the same electric field, the magnitude of the change in PE will be the same, and since the electron is -, the signs will be opposite.

Example 10: A battery has an *electric potential* of 14 volts at its *positive terminal* and 2 volts at it's *negative terminal*. It is connected to parallel metal plates that are 3 millimeters apart and insulated from one another.



a.) From what you know about the voltages, draw in the *electric field lines* between the plates.

b.) How big is the *electric field* between the plates?

Traversing from the upper plate to the lower plate (i.e., from the higher voltage to the lower voltage plate ALONG THE *E-FLD* LINES, we can write:

$$\vec{E} \bullet \vec{d} = -\Delta V$$

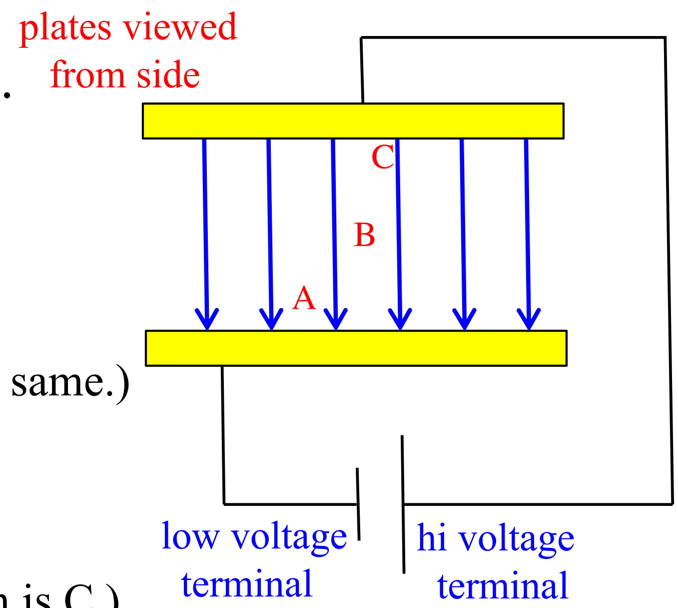
$$\Rightarrow \left|\vec{E}\right| \left|\vec{d}\right| \overset{1}{\cancel{\cos 0^{\circ}}} = -\left(V_{-} - V_{+} \right)$$

$$\Rightarrow \left|\vec{E}\right| \left(3\text{x}10^{-3}\ \text{m}\right) = -\left((2\ \text{V}) - (14\ \text{V})\right)$$

$$\Rightarrow \left|\vec{E}\right| = 4000\ \text{V/m} \quad (\text{or } 4000\ \text{N/C})$$

c.) Points A, B and C are identified between the plates.

A *positive charge* is placed successively at each point:



i.) At which point will the charge experience the greatest *electric field*?

(The *E-fld* is *constant* between the plates—all points are the same.)

ii.) At which point will the charge experience the greatest *electric potential*?

(The *voltage* closest the positive plate will be highest, which is C.)

iii.) At which point will the charge experience the greatest *potential energy*?

(A charge's potential energy at a point is related to voltage as $U = qV$, so for a *positive charge*, that will be greatest at C . . . or as close to the 14 volt plate as possible.)

d.) Now, a *negative charge* is placed at each point:

i.) At which point will the charge experience the greatest *electric potential*?

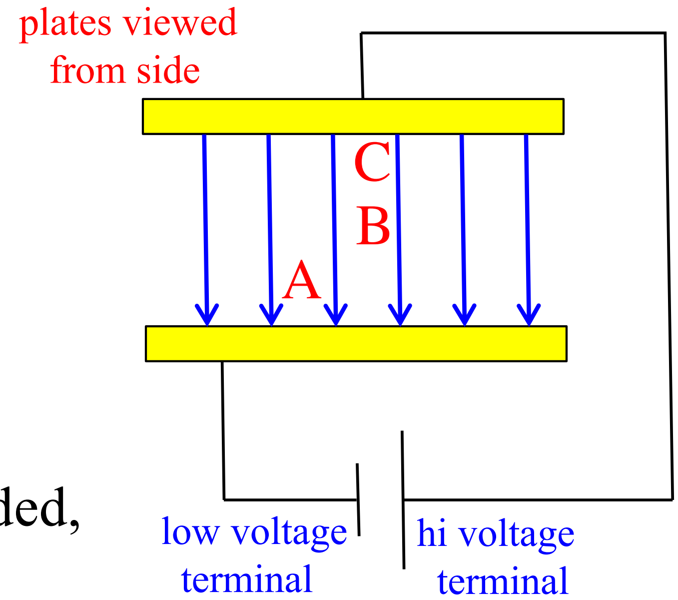
(*Electrical potential* has NOTHING TO DO with the charge feeling the effect: it's still C.)

ii.) At which point will the charge experience the greatest *potential energy*?

(Using $U = qV$, *sign included*, the greatest potential energy point for a *negative charge* is A. This makes sense if you think about it. A -1C charge on the *negative plate* (pt A)would be -2 joules whereas on the *positive plate* (pt C) it would be -14 joules. A is bigger (closer to zero)! Also, where, if you let a *negative charge* go, would it pick up the most kinetic energy? Certainly not if it was next to the positive plate. Definitely next to the negative plate at A!).

e.) An electron $(e = 1.6\text{x}10^{-19}\text{C},\ m = 9.1\text{x}10^{31}\text{kg})$ accelerates between the plates. How fast is it moving if it started from rest?

Note that the electron (charge –e) would accelerate from the negative to positive plate, and that the *potential energy* of a charge sitting at a point whose potential is *V* is $U = qV$ with the charge's sign included, we can write:



$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

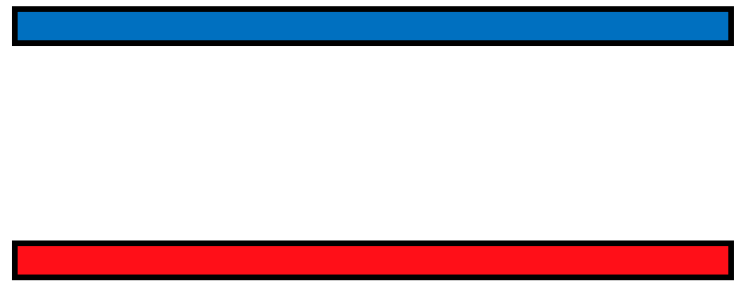
$$0 + \left((-e)V_-\right) + 0 = \tfrac{1}{2}mv^2 + \left((-e)V_+\right)$$

$$\Rightarrow \left(-1.6\text{x}10^{-19}\text{C}\right)(2\text{ V}) = \tfrac{1}{2}\left(9.1\text{x}10^{-31}\text{ kg}\right)v_B^{\ 2} + \left(-1.6\text{x}10^{-19}\text{C}\right)(14\text{ V})$$

$$\Rightarrow \quad v = 2.1\text{x}10^{6}\text{ m/s}$$

Problem 16.7 - preliminary questions

An oppositely-charged set of parallel plates 5.33 mm apart have a potential difference of 600 volts between them. An electron accelerates *downward*.



Preliminary questions:

i.) Which plate is positive?

ii.) What does the electric field look like between the plates (draw it in)?

iii.) Which plate has the higher electrical potential?

iv.) If you were to assume the higher potential plate had a voltage of 600 volts, what assumption are you making about the lower potential plate?

v.) In general, does negative charge move from higher to lower electrical potential, or vice versa?

vi.) In general, does negative charge move from higher to lower potential energy, or vice versa?

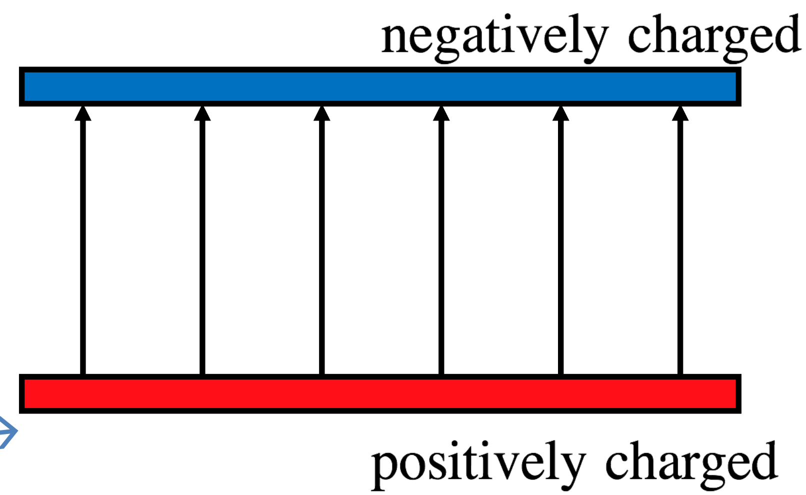
Problem 16.7 - preliminary questions

An oppositely-charged set of parallel plates 5.33 mm apart have a potential difference of 600 volts between them.

Preliminary questions:

i and ii.) Which plate is positive and does the electric field look like between the plates?

See sketch→



iii.) Which plate has the higher electrical potential?

The positive plate always has higher voltage - E fields point from high to low voltage..

iv.) If you were to assume the higher potential plate had a voltage of 600 volts, what assumption are you making about the lower potential plate?

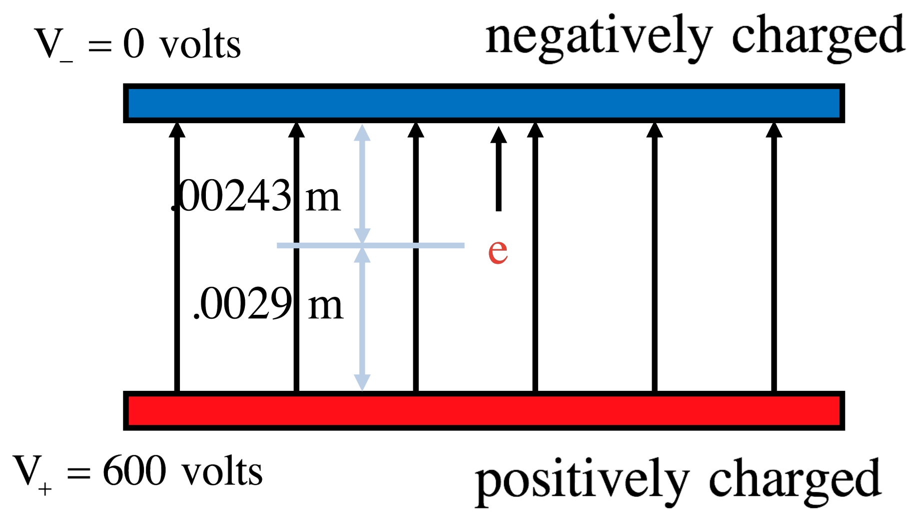
That it has a relative voltage of 0 compared to the higher potential plate.

v and vi.) In general, how does negative charge act?

Negative charges move opposite electric field lines, so they must move from the lower voltage plate to the higher voltage plate -- opposite the way a positive charge would move. For the electron, though, this will be from higher PE to lower PE (all charges naturally flow from higher PE to lower).

Problem 16.7 modified

An oppositely-charged set of parallel plates 5.33 mm apart have a potential difference of 600 volts between them.



a.) *What is* the E field magnitude between the plates?

$$\Delta V = -Ed$$
$$(0\text{ V} - 600\text{ V}) = -E(0.00533\text{m})$$
$$E = 1.126\text{x}10^5\text{ V/m (or N/C)}$$

b.) *What is* the force on an electron when sitting between the plates?

$$F = qE = (-1.602\text{x}10^{-19}\text{C})(1.126\text{x}10^5\text{N/C})(\hat{j}) = 1.8\text{x}10^4(-\hat{j})\text{ N}$$

c.) *How much* work to move the electron to the negative plate from 2.9 mm from the positive plate?

You'll have to do work to force the electron to the negative plate. If you start where the electron is shown in the sketch and proceed upward, we can write:

$$W = -(q\Delta V) = -qEd$$
$$W = -(-1.602\text{x}10^{-19}\text{C})(1.126\text{x}10^5\text{N/C})(000243\text{ m})$$
$$W = 4.4\text{x}10^{-14}\text{J}$$

15.4) The following information is known about the constant electric field shown in Figure II to the right and below: the electric field intensity is 80 nts-per-coulomb; the voltage V_A = 340 volts; the voltage V_E = 320 volts; distance d_{AB} = .25 meters; the distance d_{DE} = .50 meters and is perpendicular to the electric field; and Point C's vertical position is half-way between A and B.

a.) Is V_A greater or less than V_B?

b.) Determine the distance d_{AD} some other way than just eyeballing it.

c.) Determine V_B.

d.) There are a number of ways to determine V_C. Pick two ways and do it.

e.) How much potential energy will be available to a 6 μC charge when placed at Point A?

f.) How much work per unit charge is done by the field as a 6 μC charge moves from Point A to Point E?

g.) How much work is done by the field on a 6 μC charge that moves from Point A to Point B?

h.) If V_A had been 340 volts and V_B had been 290 volts:

i.) What would the electric field's direction have been?

ii.) What would the electric field's magnitude have been?

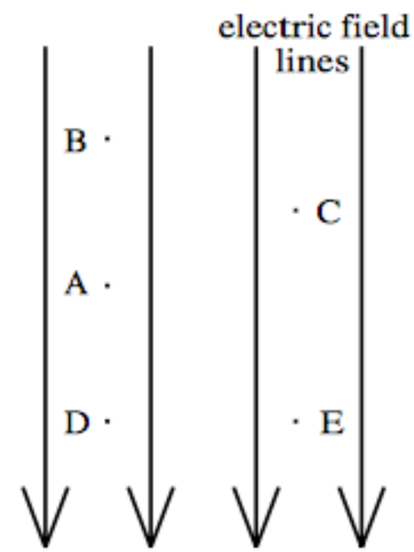


FIGURE II

15.4 answers

(a) Less than V_B

$$(b)\ \Delta V = -Ed, \qquad d = -\frac{\Delta V}{E} = -\frac{320V - 340V}{80} = 0.25\ m$$

$$(c) \Delta V = -Ed \rightarrow \quad (340\ V - V_b) = -\left(80\,\frac{N}{C}\right)(0.25\ m) \rightarrow\ V_B = 360\ V$$

$$(d)\ \Delta V_{BC} = -Ed, \qquad V_C - 360\ V = -\left(\frac{80N}{C}\right)\left(\frac{0.25m}{2}\right) \rightarrow V_C = 350\ V$$

$$(e)\ U_A = qV_A, \qquad U = (6x10^{-6}\ C)(340\ V) = 2.04x10^{-3}\ J$$

$$(f)\ \frac{W}{q} = -\Delta V_{AE} = -(320V\ - 340\ V) = 20\ J/C$$

$$(g)\ W = -q\Delta V_{AB} \quad W = -(6x10^{-6}C)(369V - 340V) = -1.2x10^{-4}\ J$$

(h) (i) Field would reverse direction (lines go from high→low voltage

(ii) $\Delta V = -Edcos\theta, \qquad (290V - 340V) = -E(0.25m)cos0 \rightarrow \quad E = 200\ N/C$

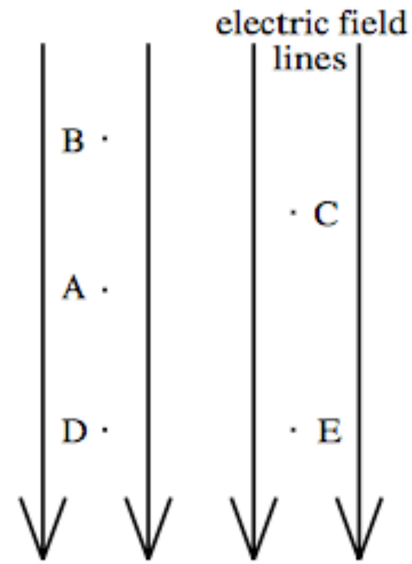
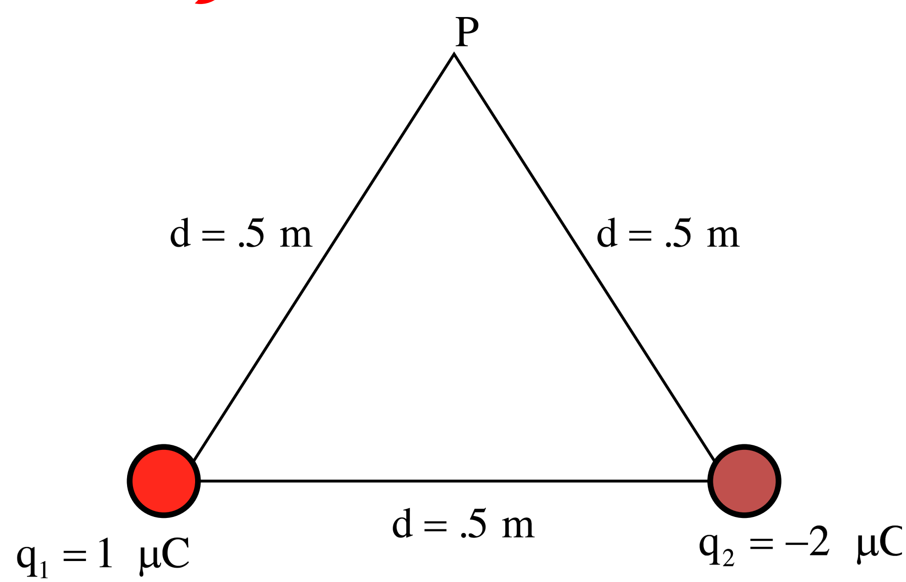


FIGURE II

Pt charge potential example - 16.60



a.) What is q_1's electrical potential at P?

$$V_1 = k\frac{Q}{r}$$
$$= \left(9x10^9\ V \bullet m/C\right)\frac{\left(10^{-6}\ C\right)}{\left(.5\ m\right)}$$
$$= 1.8x10^4\ \text{volts}$$

b.) What is q_2's electrical potential at P?

$$V_2 = k\frac{Q}{r}$$
$$= \left(9x10^9\ V \bullet m/C\right)\frac{\left(-2x10^{-6}\ C\right)}{\left(.5\ m\right)}$$
$$= -3.6x10^4\ \text{volts}$$

c.) What is the total electrical potential at P?

$$V_1 + V_2 = 1.8x10^4\,\text{volts} - 3.6x10^4\,\text{volts}$$
$$= -1.8x10^4\,\text{volts}$$

d.) What is the work required to move a 2 μC charge from infinity to P?

As the voltage at infinity is zero, we can write:

$$W = -q\Delta V$$
$$= -(q)\left(V_P - V_\infty\right)$$
$$= -\left(2x10^{-6}C\right)\left[\left(-1.8x10^4\,\text{volts}\right) - 0\right]$$
$$= 3.6x10^{-2}\ \text{joules}$$